G.H. Hardy: The Leading Mathematician in England

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The following is the approximate script I used. The content (and jokes) may differ from those that were in the actual talk.

Good afternoon everyone, and thank you to the TMS for inviting me to speak. (This may be the first time anyone has called me "distinguished".) It is my honour today to talk about our founder, G.H. Hardy.[1]

This talk will focus on Hardy's contributions to the mathematical community beyond what we might term "mere research".

1 Who was Hardy?

Since you're at Cambridge, I imagine you probably think you already know a fair amount about Hardy. Who here has read *A Mathematician's Apology*?[2]

Okay, for the three people who haven't: Hardy was a prominent (indeed, probably *the most prominent*) mathematician at Cambridge (and therefore in England) in the early twentieth century.

I shall begin with some general biographical remarks, so you might have a fuller picture of the man.[3]

[1] A note on nomenclature: during his life, almost everyone refered to Hardy as "Hardy". His closest friends called him "Harold". No one called him "Godfrey", for reasons that will become apparent.

[2] [9]

[3] Most of the biographical information we have on Hardy comes from Titchmarsh's obituary (reprinted in Hardy's *Collected Papers*, Volume I) and Snow's essay on Hardy in [16], which overlaps considerably (albeit with different emphasis) with his foreword to the 1967 edition of *A Mathematician's Apology*, and the more cricket-focussed [15], which is reprinted in [1]. Other scraps are found in Littlewood's famous book [12]. Currently the best overall (although non-technical) biography of Hardy is actually Kanigel's of Ramanujan [11].

He was born in 1877 in Surrey, the son of two Lincolnshire schoolteachers. He went to boarding school at Winchester, and was one of those people who absolutely hated it. He was completely constitutionally unsuited for it, almost died one winter, and rebelled against everything it signified for the rest of his life, including religion, the establishment, and hot roast mutton. His twin joys were Mathematics and Cricket, and he felt the school ruined his abilities in the latter by poor coaching. At this point, he enjoyed Mathematics primarily competitively: it was the one thing that he could beat anyone at, even at school. I imagine some of you know the feeling.

He came up in 1896, and experiences a crisis of motivation, which we shall come back to in a minute. For now, let us continue with overview.

His working day in the summer generally consisted of working in the morning, and he would then walk to Fenner's to take in an afternoon's cricket. (He played cricket and real tennis himself.) He devoured scorecards in the morning papers, delighting in their numerical richness.[4] One sees in some ways the pinnacle of this in his "gas" for the first paper on what became known as the Hardy–Littlewood maximal function:

> This phenomenon is best exhibited in terms of cricket [...]

General opinion was that he was uncommonly good-looking, although he couldn't stand his own appearance. He would cover the mirrors when staying in a hotel room.

He was eternally suspicious of technology. "Technology" has quite a broad meaning here: he apparently never used a fountain pen. He never wore a watch, and found telephones profoundly suspicious (C.P. Snow recalls him once saying "If you *fancy yourself* at the telephone, there happens to be one in the next room"[5]). He communicated, in order of preference, by postcard, letter, or in person, with the telephone a distinct last.

He was profoundly shy in public: he wouldn't acknowledge friends in the street, and didn't shake hands.

He was obsessed by rating people, in various categories. For many years he regarded Jack Hobbs (of Surrey) as the zenith of batsmanship,[6] and compared everything to him: the crème de la crème was described as being "in the Hobbs class".[7] This was rather upset when he saw Bradman bat for the first time.

[4] Hardy was completely bewitched by integers. No wonder he took to Ramanujan.

[5] Snow, somewhat to his discredit, seems to have been incapable of much more than a certain level of fictionalisation in his novels: most of his characters are fairly thinly disguised versions of his aquaintances. Hardy is found in *Homecoming*, fictionalised as the art critic Austin Davidson. Indeed, this very quotation is inserted with little alteration on the first page of Ch. 37.

[6] For many, he remains the zenith of English batsmen.

[7] Even unto his deathbed, where he had an argument with Snow about whether Vinoo Mankad was an all-rounder of the Rhodes or Faulkner class. (He was, by the way: for many years the Indian cricket team was "Mankad, plus some other blokes".)

Hardy's most constant characteristics is his rigorous assessment of anyone and anything, especially himself: he gave himself a mere 25/100 as a mathematician.[8]

I've spent the last 5 minutes making Hardy out to be profoundly eccentric. To put this in context, Norbert Weiner noted that, unlike his *alma mater* Harvard, at Cambridge eccentricity was positively encouraged: one had to develop some peculiarities to be taken seriously.[9]

2 Tripos reform

Okay, so in short, Hardy's a mathmo. He's just like most of us. Let's rejoin him in Cambridge as a first-year.

Hardy arrives in Cambridge in 1896. At the time, the Tripos is the most arduous, difficult and competitive mathematics examination that the world has ever seen. The results were read out in Senate House, and published in the national and local newspapers. One could purchase commemorative photographs of the Senior Wrangler and Proxime Accessit. The exams themselves were three hours each, two to a day, for many days (roughly 14 exams in two weeks in the early twentieth century). If you wanted to do well, you employed the services of a coach. It was not uncommon for highly-placed students to have nervous breakdowns.

But, and this may not surprise you, even at the time the Tripos was widely regarded as almost absurdly out of date. It contained almost nothing that you would recognise as university mathematics. This meant that Cambridge was, in terms of pure mathematics, completely backward. Indeed, although the fortunes of Applied Mathematics at Cambridge seemed rather better, with a string of luminaries you may have heard of such as Kelvin, Maxwell, Heaviside, &c, by Hardy's day even the parts of the Tripos that we would call Applied Mathematics were, to put it mildly, somewhat behind the times: Russell (who took the Tripos in 1893) reports that at no point in his undergraduate studies of electricity was he introducted Maxwell's equations.[10][11] Undergraduates were not required to learn modern mathematics, their teachers knew little of modern mathematics, and almost nothing was done in the way of "research". And since most professors of mathematics in England were educated at Oxford or Cambridge at the time, the same was true of most other English universities.

Hardy was regarded as of exceptional talent, and Trinity persuaded him, as an experiment, to take the Part I exams at the end of his second year.[12] This was the first time this was ever

[8] Also on this list: Littlewood (30), Hilbert (80), Ramanujan (100).

[9] So nothing like today, then.

[10] "And," Hardy couldn't help adding with perhaps a small amount of irony "I have also been told by friends whom I believe to be competent that Maxwell's equations are really rather important in physics."

[11] The accuracy of Russell's recollections of this period may be called into question: his Autobiography is notoriously inaccurate and incomplete: it doesn't even mention Hardy at all!

[12] One other student was also part of this experiment: James Jeans. See, e.g. Milne's biography, [13, pp. 4ff.]

attempted, and even with exceptional students, it was expected that they would likely rank no better than 15th. There is speculation that this was a sign of the growing expectation of reform: the exam would surely need adjustment if the best candidates could take it after two-thirds the expected time!

But despite Hardy enjoying mathematics competitively at school, the barren territory of the Tripos syllabus appalled him: within a couple of weeks, he was considering switching to History.[13] Thankfully, this did not happen.

What stopped Hardy from abandoning Mathematics for History? As stories about Hardy go, this is one of the better-known ones: A. E. H. Love[14] hands him a copy of what would become one of the most influential textbooks of the late nineteenth century: the second edition of Jordan's *Cours d'Analyse*. It is no exaggeration to say that this book was a profound influence on an entire generation of mathematicians: one needs only mention Borel, Lebesgue, and Hardy himself. It covers Analysis, in the sense we mean, from basic epsilontics to what was then the research frontier: what we now call Jordan content and Jordan's curve theorem.

Ah-ha! Suddenly, Hardy finds the Mathematics he needs: he devours the *Cours d'Analyse*, and devotes himself to Analysis forevermore.

Hardy (and Jeans) takes the exams in his second year as planned. The experiment is a great success:, Hardy placed fourth,[15] good enough to be given a Fellowship at Trinity.[16]

By now, the winds of change were blowing. Certain Cambridge mathematicians had begun to make significant contributions to Analysis. Indeed, one, W. H. Young, whom you'd never heard of, almost made the most important discovery of twentieth-century Analysis.[17] The other figure of the period that we should mention is E. H. Hobson, who had for some years been crusading for Tripos reform, and would go on to write in many ways the definitive book in English on the theory of functions of a real variable.[18] But this was for the future.

[13] *Plus ça change, &c.*

[14] Who makes a brief appearance in our Waves course these days.

[15] Which disappointed him forever after: as Littlewood said, the game was stupid, but he still felt he should have won it.

[16] Jeans came even above Hardy, in joint second.

[17] Now, I know this is not relevant, but I must tell you something about Young. He was a Cambridge undergraduate, then Tripos tutor, but then was seduced away from Cambridge by continental mathematics, and spent much of the rest of his career at GÄ¶ttingen. Remarkably, he struck out on a sparkling research career in his thirties, accompanied by his equally formidable wife, Grace Chisholm Young, who you really, *really* should know about, wrote several of the first books on Analysis, topology and the real numbers in English. What was this major discovery Young almost made? He was one of three people who near-concurrently discovered a new theory of integration. Indeed, once he realised that he had been anticipated, he was the first to concede priority, and christened the new theory "Lebesgue integration". So Cambridge did produce some brilliant analysts, but they didn't live in Cambridge.

[18] At the time. The theory had developed so much by 1926 that it necessitated an almost complete rewriting of the book.

Hardy's early research work in Mathematics was, by his own estimation, of little value.[19] But as yet it was not Hardy's research that was important. From the very first he strove against the Tripos. Perhaps consciously emulating the Analytical Society in the 1820s, he had realised that there was one proven way to change the Tripos: to examine for it (and hence have the questions in your power), and to write the book that was used for it.

A committee was formed to reconsider the Tripos, with Hardy as its secretary, and a number of the other names I have mentioned on it, along with various other names you would recognise. It produced three reports in 1906, recommending the complete rewriting of the examination, a significant reduction in the number, and alterations to the schedules. Each was fiercely debated in the Senate, and in the national newspapers: many prominent ex-Cantabs wrote spluttery, not-in-my-day letters protesting about standards, academic excellence and the decline of the University.[20] Most vociferous, not surprisingly, were the professional tutors: one remarked that abolishing the Order of Merit would be akin to "publishing the results of the Derby in alphabetical order", which itself says much about the general attitude to the Mathematical Tripos exams at the time.

But the Senate eventually yielded, and passed a Grace fulfiling the recommendations, and abolishing the Order of Merit.

There remained the small issue of textbooks. The whole point of the new regulations was to bring the Tripos up to date, and in particular, to bring in some of the vast amount of Analysis that had taken over Continental mathematics in the latter half of the nineteenth century. So Hardy begins his next great project: a textbook suitable for the new Tripos.

2.1 Not quite a *Cours*: *A Course of Pure Mathematics*

Some will tell you that Hardy writes the first Analysis book in English. This is very much not true. Indeed, much of the point of Hardy's text was to take the able undergraduate from his schooling in algebra and trigonometry to being able to read such works as did exist in English at the time. Let's see why these were not suitable for exposing undergraduates to.

In 1902 Whittaker's book *A Course of Modern Analysis* appeared, which treated the theory of special functions rigorously. Alas, it contained one of the most opaque and useless introductory chapters in the history of mathematical exposition, providing no help whatsoever to the naÃ¨ve ingenue who hoped to begin at the beginning.[21][22] Not that this is a problem any more, of

[19] Recall what we said about Hardy's own ratings.

[20] Suffice to say, as in most such cases, that many of these comments were complete rubbish. Hardy himself gives a rebuttal in *The Case Against the Mathematical Tripos*.

[21] My PhD supervisor once encouraged me to start in the middle of a book "If you start every book at the beginning, you'll probably die before you get to the bit you need".

[22] English logician Jourdain is especially scathing about it in his joint review [10] of the first and second editions.

course.

Another noted example was the first edition of Hobson's *Theory of Functions of a Real Variable and the Theory of Fourier's Series*, which had the misfortune of being written at the same time Lebesgue was completely revolutionising the theory.[23] It too is completely intractable to the beginner: the first three chapters walk through much of the cutting edge of set theory and topology of the time: ordinals, cardinals, measure theory, all just what you want before you meet limits for the first time.

Existing undergraduate texts at best dealt in what we would call Calculus, entirely unsuitable as an introduction to these books. Something was desparately needed to fill the gap.

Perhaps closest to what Hardy was aiming to do was (once and future Cambridge mathmo) Bromwich's *An Introduction to the Theory of Infinite Series*. This is a fascinating and deeply flawed book. It was possibly the first approximately–undergraduate level course in rigorous Analysis published in England. One only has to open the book and look at the table of contents to see what happened: Bromwich was trying to write a book about infinite series, to a certain level of rigour. At some point, he realised that to do so, one needs to include something about convergence. And it's also good to have the logarithm function, for the purposes of establishing the logarithmic scale if nothing else. And if you've got the logarithm, you may as well do the trigonometrical and cyclometrical functions as well. So he ended up with a decent-sized book on infinite series, with a colossal set of appendicies covering many other aspects of what you do in Analysis I. This is clearly not going to work as a textbook either.

So Hardy sits down and writes a book. Here it is:[24] the first decent English Analysis textbook. It was published in 1908, just in time for the new regulations. The content, real numbers, limits, continuity, derivatives, integrals, will not strike you as surprising. But that's because you already think you know what such a book should look like. This is the first time, at least in English, that these particular topics are placed together in this way.

But the subjects covered are not the most interesting part. To look at the first edition of *A Course of Pure Mathematics* is to have a fascinating insight into Hardy's mindset at the time. At this point I can no longer resist giving some choice quotations from the Preface, where Hardy's aim is put most clearly and desk-poundingly:

> […] I have, here and throughout the book, kept geometry in a strictly subordinate position and used it merely for purposes of illustration. […]
>
> The notion of limit is one that has always presented grave difficulties to mathem-

Even in its fourth edition (1927), the opening chapter is best ignored.

[23] Inevitably, the second edition is essentially a different book altogether, having to be rewritten from the ground up.

[24] [7]

atical students even of great ability. It has been my good fortune during the last eight or nine years to have a share in the teaching of a good many of the ablest candidates for the Mathematical Tripos; and it is very rarely indeed that I have encountered a pupil who could face the simplest problem involving the ideas of infinity, limit, or continuity, with a vestige of the confidence with which he would deal with questions of a different character and of far greater intrinsic difficulty. I have indeed in an examination asked a dozen candidates, including several future Senior Wranglers, to sum the series $1 + x + x^2 + \ldots$, and not received a single answer that was not practically worthless—and this from men quite capable of solving difficult problems connected with the curvature and torsion of twisted curves.

I cannot believe that this is due solely to the nature of the subject. there are difficulties in these ideas, no doubt: but they are not so great as many other difficulies inherent in mathematics that every young mathematician completely overcomes. The fault is not that of the subject or of the student, but of the text-book and the teacher. It is not enough for the latter, if he wishes to drive sound ideas on these points well into the mind of his pupils, to be careful and exact himself. He must be prepared not merely to tell the truth, but to tell it elaborately and ostentatiously. he must drill his pupils in 'infinity' and 'continuity', with an abundance of written exercises and examples, as he drills them at present in poles and polars or symmetric functions or the consequences of De Moivre's theorem. Then and only then he may hope that accurate thought in connexion with these matters will become an integral part of their ordinary mathematical habit of mind. It is this conviction that has led me to devote so much space to the most elementary ideas of all connected with limits, to be purposely diffuse about fundamental points, to illustrate them by so elaborate a system of examples, and to write a chapter of fifty pages without advancing beyond the ordinary geometric series.

[...]

Anyone who has read this book will be in a position to read with profit Mr Bromwich's *Infinite Series* [...]

[...] I am on the whole of opinion that, if any proposition is too difficult to be proved properly, its statement and application had better be postponed. I am well aware that there is much to be said on the opposite side. A very plausible case can be made out for the habitual exercise of the student in the application of results whose proof is too difficult for his full comprehension. But I have found that I

> cannot myself write a book on those lines: nor am I fully convinced that such exercise is either necessary or desirable. [...]

Everything I have quoted was excised in later editions, and in the Preface to the Seventh edition in 1938,[25] a somewhat embarrassed Hardy quotes Littlewood as describing his vociferous prose of the First edition as "a missionary talking to cannibals".[26]

The reviews were extremely favourable (although it should be noted that at least some were written by Hardy's co-reformers).[27] But at the end of the day, the reviews are rather less important than the results, which will bring us to the next topic.

Before we quite move away from Tripos, however, we should note that Hardy remained an enemy of Tripos, even in the watered-down reformed version. He lays out his arguments in his address to the Mathematical Association in 1926, "The Case Against the Mathematical Tripos".[28] He believed that "honours" examinations such as the Tripos, even in watered-down form, were useless and damaging. But he took no more part in Tripos reform after 1910, concentrating his attentions elsewhere.

Still, being present in the Tripos gives Analysis a foothold in England. Now let us look at Hardy's later promotion of Analysis in England.

3 The growth of Analysis in England

In 1900, it is fair to say that essentially no Analysis was taught in England at all.

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[25] [8]

[26] Thankfully he understood that, as more conventionally-written Analysis books appeared in English, there was no point in watering down the *Course* to appeal more widely, so the later editions are mostly resettings with corrections and expansions rather than wholesale rewritings, and we are left with many of the fascinating eccentricities of the original.

[27] For example, Jourdain [10] says

> [...] Mr. Hardy seems free from any tendency to insert notes mentioning that there is a difficulty which do not say what it is; this process is not unknown among unpractised teachers, and only succeeds in quite pointlessly puzzling, irritating, or even boring a student.
>
> [...]
>
> Every page breathes Mr. Hardy's supreme competence as teacher and mathematician.

Berry [2] says

> [...] when Mr. Hardy sets out to prove something, then, unlike the writers of too many widely read text books, he really does prove it. Probably a really acute critic reading the book would discover here and there gaps in the logic, but I feel sure that these would prove to be few and trivial; and if the book is widely read, I for one shall hope to avoid in the future the many weary hours that have usually to be spent in convincing University students that "proofs" which they have laboriously learned at school are little better than nonsense.

[28] Available in *The G. H. Hardy Reader*, [1]

By 1928, Hardy was able to address the LMS in his Presidential Lecture, delighted at having moulded its *Proceedings* into a publication focussed almost exclusively on Pure Mathematics, and particularly Analysis and inequalities.

This change in circumstances was largely due to the work of Hardy himself: firstly, he had literally written the book on Analysis that was deployed in university teaching. At first it was Hardy or nothing, so naturally a generation of mathematicians grew up on Hardy in England, just as a generation of largely French mathematicians grew up on Jordan. The latter were responsible for the great turn-of-the-century revolution in analysis. While we can't say the same of the former,[29] it was certainly now true that the new generation were not only growing up to become mathematicians, but to become Hardy's sort of mathematicians.[30]

Hardy's second line of influence was rather more direct: he wrote and published a huge amount, both alone and in collaboration with others. To give you some idea, Hardy's 350-odd papers were published in 7 volumes, and the majority of these were written between 1910 and 1935. The most fruitful of these partnerships were with Ramanujan and Littlewood, the latter being probably the most productive collaboration in the history of mathematics, responsible for some 100-odd papers.

Bohr famously said that there were three great English mathematicians, Hardy, Littlewood and Hardy–Littlewood. You are probably expecting me to say something about the third of these. Hardy and Littlewood began collaborating in 1912, when both lived in Trinity. Okay, but Hardy moved to Oxford in 1920, so surely they slowed down then? Quite the opposite: because Hardy and Littlewood communicated mostly by written notes, even when they lived mere yards from each other,[31] it was quite simple to continue collaborating while in different institutions. They covered many topics, all linked by the common theme of Analysis, with quite a lot of Analytic Number Theory. The 1910s and 20s were in many ways the golden age of Analytic Number Theory, and Hardy and Littlewood, separately or together, were right at the forefront. Sadly I don't have any more time to go into more detail about this collaboration, so let's move on.

Thirdly, he was for much of the 10s and 20s the Secretary of the London Mathematical Society, and was able to take advantage of this in several ways: to forge international relationships and research collaborations (of which more later), and to use its Proceedings to publish work arising from these, in addition to his own considerable output.[32] But even this was not enough:

[29] Although we would like to point out that one P.Daniell, the last Senior Wrangler under the old system, later discovered an integral equivalent to Lebesgue's, and indeed more suited to the functional analysis the integral was later adapted to.

[30] This influence was acknowledged by two of Hardy's obituaries, one of which was written by his pupil Titchmarsh, and reprinted in Volume I of Hardy's *Collected Papers*.

[31] Mathmos ...

[32] To give a decent idea of the impact Hardy had in the *London Mathematical Society*'s journals, we only need look

to satisfy the need he saw for more journals devoted to Pure Mathematics, he was a major force in launching two new journals, the *Journal of the London Mathematical Society*, for shorter papers, and a revival of one of the two journals that Glaisher had edited almost singlehandedly for much of his life: the *Quarterly Journal of Pure and Applied Mathematics*, revived as the Oxford-based *Quarterly Journal of Mathematics*. Both survive, and indeed thrive, to this day.

Lastly, Hardy essentially created his own school while at Oxford. Before, Oxford had had little in the way of prominent mathematicians, but Hardy spent much of his ten years there as advisor to many graduate students who would go on to prominence in Mathematics, and in particular in Analysis: we need only mention Titchmarsh, Mary Cartwright (later Dame Mary Cartwright), and Wright.[33]

Hardy was a brilliant advisor, and one of his better-known creations was his conversation class, where the community of mathematicians at Oxford would gather to discuss a paper one of them would present. Titchmarsh said it was "a model of how such a thing should be run": it was sufficiently informal that people could actually engage in useful mathematical conversations. Contrast with seminars today, where the speaker gives a nigh-incomprehensible presentation, followed by zero questions from the audience.

(When Hardy returned to Cambridge in 1931, he kept this going, originally hosting it with Littlewood. But Littlewood quickly got fed up with what he saw as Hardy quibbling over details when he was trying to present an overview, so it rapidly became "The Hardy–Littlewood seminar at which Littlewood was never present".[34])

The culmination of Hardy's success was in his election to the Presidency of the LMS in 1926, and his continuity and influence was epitomised by his boast that he had never missed a single meeting since he became Secretary in 1917.

Not since Newton had any English mathematician had such an influence on the mathematical culture of the country. And it's not much of a stretch to speculate that never will it happen again. Hardy had seized his opportunity, and very much remade British mathematics in his own image.

But though Hardy and Newton were alike in this and some other respects, there were key differences: unlike Newton, Hardy was, in his own way, gregarious, outgoing, and forged vital relationships abroad.

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at the numbers: between 1900 and 1940, the most prolific contributors were Burnside (31 articles), Hobson (32), Young (69) and Hardy, with an absurd 106 (56 sole author, 50 with Littlewood), which is nearly 10% of the whole lot over this period. And remember that Hardy slowed down later. These figures come from [14].

[33] With whom he wrote perhaps his other textbook, *An Introduction to the Theory of Numbers*. This text is so highly regarded I have known mathmos to swear oaths on it.

[34] This is related by Dame Mary Cartwright in [17]: she appears to have enjoyed disparaging Hardy, both to his face and otherwise.

4 International relations

4.1 The First World War

In spring 1914, Ramanujan came to England thanks to the efforts of Hardy and various other familiar names at Trinity. Brilliant and fascinating though this period is, and a highlight of Hardy's career,[35] I do not have the time to go into useful detail about it today.

As we all know, in late 1914 the First World War broke out. As you also can no doubt guess, this caused considerable problems for mathematicians. A great many friendly collaborations were severed, many permanently. In general the intellectual communities in England and Germany had been on excellent terms, and Hardy and most of his contemporaries were by nature Germanophiles.[36]

But they still had to go to war. Well, most of them. Russell was a famously vocal conscientious objector. Hardy was not, and did volunteer for service, but was rejected on medical grounds.[37] So he remained in a fractious Cambridge. As seems to happen under such circumstances, a distinct current of bellicosity emerged even in the somewhat rarefied atmosphere of the College. Hardy found this both bewildering and hurtful, and one suspects that without the distraction of Ramanujan's extraordinary outpourings to contend with and mould into publishable material, quite apart from their own fruitful collaboration, Hardy would have found the experience genuinely unbearable.

But not all communication ceased between mathematicans in enemy nations. A key conduit was provided by the leading Swedish mathematician of the time, Mittag-Leffler.[38] Indeed, he had form in this regard: he had actually founded the journal *Acta Mathematica* after the Franco–Prussian War in 1882 to try to maintain relations between French and German mathematicians:[39] he deliberately acquired and published excellent work from the most well-known on both sides, and published it side by side. Now he saw the opportunity to do the same again, and devoted two volumes in 1916 to this idea.

Meanwhile, Hardy continues publishing. Okay, that's not saying much. But Hardy continues publishing in *foreign journals*. This was obviously not easy at the time! He actually publishes 9 papers in foreign journals during the War. Suffice to say that no other English mathematician

[35] Hardy himself would say *the* highlight

[36] Feelings were reciprocated; indeed, it often seemed that Hardy was more appreciated in Germany than his own country: the quote in my abstract comes from a letter from Hilbert to the Master of Trinity, complaining that they were not accommodating him suitably: being the best mathematician not only in Trinity but in England, he should naturally have the best rooms in college. One wonders how well this went down.

[37] This was probably for the best: one doubts that Hardy's delicate constitution would have survived for long, whatever conditions he was placed in.

[38] Yes, Mittag-Leffler, like Swinneton-Dyer, is one dude.

[39] Sweden being neutral in this War, as it was again in the First World War.

has anything like this foreign presence.

One consequence of this refusal to break off communication was that by the end of the War, Hardy was easily the best-known English mathematician in Europe.

4.2 The Post-War Conflict

Finally in 1918 the War ended. Europe was left in a sorry state, the old institutions devastated, an entire generation destroyed or scarred forever. Not surprisingly, there were many who had little interest in readmitting German science to the fold. While a minority, many of them had loud voices.

I apologise for the content of what I am about to read. I would like to say it comes from a less enlightened age, but I'm honestly not sure how true that is. What I do not apologise for is reading it: you should know that this happened.

They would write ghastly letters to Nature and the Proceedings of the Royal Society,[40] declaring that

> […] for the next twenty years at least all Germans will be relegated to the category of persons with whom honest men will decline to have any dealings.

Even worse are the ones that attempt to rationalise their hatred: *Nature* reports on American studies into "the mental character of German scientists", unsurprisingly finding them lacking in originality and creative power, relying instead on "intensive self-advertisment".

There is perhaps no simpler indicator that sometimes, complete crap gets published.

In this atmosphere, Hardy writes to Mittag-Leffler asking his advice: should he and Littlewood try to get something published in a German journal? Mittag-Leffler says that this is only likely to lead to the press writing "burning polemics". Again, how times have changed! Hardy and Littlewood went ahead regardless, and the paper appeared in 1920. History does not record any burning polemics following.

Mittag-Leffler encourages what we might term perhaps a little unkindly a campaign of mutual buttering-up: publishing old letters between esteemed French and German scientists in which each praise the other's work.

Hardy's approach was rather different. First, he was absolutely desperate to visit his fellow mathematicians, if only to gauge their attitudes to future international collaboration. And second, he confronted the obstacles head-on.

The growing exclusionist movement infiltrated the committees of the international organisations. Perhaps the most infamous example was that of the sixth International Congress of Mathematicians. This had been planned to take place in Stockholm in 1916, but the War

[40] This, and most of the other quotations in this section, are taken from letters cited in Dauben's collection [5].

got in the way. At the end of the war, the council, which happened to be mostly composed of Frenchmen, had other ideas. They decided to instead hold it in Strasbourg (yes, the city in Alsace moved from Germany to France after the end of the War), and exclude all German mathematicians.

Hardy, and many others, were appalled, and boycotted the conference: indeed, it has the lowest attendence of any ICM on record. Pressure from Mittag-Leffler and others had the official name of the conference changed to the International Congress of Mathematics (because it could hardly be said to be an international congress of mathematicians if a whole nation was excluded).[41] The status of this conference, and the next, also Germanles one in Toronto, remained so controversial that to this day the ICMs do not have numbers, just so we don't have to decide if Strasbourg 1920 and Toronto 1924 count.

Similarly, there are efforts to set up new "international" mathematical and scientific unions with explicit rules about excluding Germans, and even neutrals. Hardy wrote an icy response to a letter to the *Times* calling for such things, in which he carefully shredding their position.

One picturesque moment in this saga comes from a letter Hardy writes to Mittag-Leffler after attending a conference in Jena (in Germany, apparently). He had just been holidaying in Cornwall (probably with Littlewood: this did actually happen occasionally), and was on his way to Stockholm. After travelling by train and boat (and Hardy famously hated boats) the hundreds of miles to Copenhagen, he met Harald Bohr at the station, who said "I'm on my way to a conference in Jena. You should come!" Hardy is not enormously impressed with this, having been travelling for nearly four days already, and his luggage being already on its way to Stockholm, but

> […] I felt that, having regard to my own relations with the Germans, and the great importance of showing all possible friendly feeling towards them, the invitation amounted almost to a "royal command", if it were physically possible to accept it.

He duly accepts and departs for Jena immediately, and arrives completely exhausted, but has a wonderful time at the conference. As one of only two Englishmen there, he is completely spoilt by the Germans, and encouraged by the enthusiasm for research he encounters among the young mathematicians.[42]

The tail of this letter also contains the definitive statement of Hardy's views on the matter:

> For my own part I have in no respect modified my former views, and am in no circumstances prepared to take part in, subscribe to, or assist in any manner directly

[41] The French won in the end, though: the proceedings emerged under the title *Comptes rendus du Congrès international des mathématiciens*, not *mathématiques*.

[42] Hardy doesn't tell us if his luggage ever showed up.

> or indirectly, any Congress from which, for good reasons or for bad, mathematicians of particular countries are excluded.

Finally, he adds the rather damning

> You are probably aware that the new Unions have excited no enthusiasm at all in England, and that, for example, the London Mathematical Society, and the biologists generally, have declined to be associated with them. There is no doubt at all that the general opinion in England regards the "boycott" as ridiculous, and would be quite in sympathy with any attempt to break it. Unfortunately nearly everyone, though in general sympathy, is quite apathetic about it; and Schuster and a few of the other older people get their way, merely because no one has the courage or energy to oppose them.[43]

Indeed, after the departure of Russell and the departure and subsequent death of Ramanujan, Hardy found that the atmosphere in Cambridge was too much, and took a professorship at the Other Place, adopted by New College. Here he made the best of it, and spent the best years of his career, only drawn back to Trinity ten years later by the comforts of being able to remain in College after retirement. He felt much more relaxed there, and felt they appreciated him more than ever they did at Cambridge. Moreover, as previously noted, his collaboration with Littlewood really took off.

By the 1928 ICM in Bologna, the anti-German rules had at last been scrapped, and it appeared that the mathematical and scientific communities had finally made peace, and now, normal service could resume.[44] However, this supposition proved to be tragically incorrect.

4.3 The disastrous '30s

When I mention a disaster in Mathematics in the 1930s, your mind instantly springs to Gödel, right? We all know that, at the same conference in 1930 where Hilbert announced his retirement with a final appeal against *ignorabimus*, Gödel announces his first Incompleteness Theorem, and ruins Mathematics forever.

But, and I can't really emphasise this strongly enough, *no one cared*. Okay, that's not quite true. But the consequences of the Incompleteness Theorems were really not properly understood by your typical mathematician until rather later. Indeed, it is speculated that Hilbert

[43] Aping, possibly John Stuart Mill's "Bad men need nothing more to compass their ends, than that good men should look on and do nothing."

[44] There's a glorious photograph of Hadamard on the beach during the ICM in Bologna that really seems to sum this up that I'd love to show you. It's found on p. 21 of *International Mathematical Congresses: An Illustrated History 1893–1986*.

himself never quite understood just how fully his programme had been destroyed. So yes, disaster, but no one notices.

While the consequences of Gödel's results were simply misunderstood or ignored,at least initially, what shortly became impossible to ignore was the precipitous rise to power of the Nazis. This almost immediately had consequences for mathematicians: the prominent number theorist Edmund Landau was immediately forced out of his professorship at Göttingen, and as I'm sure you know, over the next few years, many mathematicians in Germany and other countries were either excluded from jobs at universities, or forced out otherwise by the worsening political situation. There were relief efforts for these mathematical refugees created in several countries; one of the first was the Academic Assistance Council founded in Britain in May 1933, which had Hardy as a prominent and active member.

By this time Hardy was back in Cambridge, and he writes

> There are several men whom I should wish to recommend very strongly—for example, Heilbronn (perhaps the best of all the mathematical refugees) and Rado. But I should wish to see them here, or at Oxford, and not in Canada and Australia.

Hardy got his wish in many cases: he was able to find 18 places at Cambridge, and find employment elsewhere for many others. This was by no means a solo effort, and we could mention many other significant names here, but Hardy's influence and recognised judgement was often a key factor.[45]

We can do worse than end with the following quote from Nobel laureate and Trinity man A. V. Hill:[46]

> Professor G. H. Hardy had been deeply concerned since 1933 with the fate of his fellow-mathematicians on the continent and had interested himself actively and generously, in co-operation with the Society for the Protection of Science and Learning, in finding places in Cambridge and elsewhere for those whom persecution had driven out. Many of these will remember him, not only with admiration for his intellectual eminence but with afection for his sympathy and succour in their emergency. Hardy in many ways was otherworldly, but in his deep solicitude for the dangers and difficulties of his colleagues he showed not only a broad humanity but a fine and resolute loyalty to the universal integrity and brotherhood of learning.

[45] We might also mention as an aside that Sommerville College at the Other Place tried to get Emmy Noether, but she ended up in America in the end instead, as did many others.

[46] [3]

In many ways, this was the most important thing Hardy ever did. He used his unique influence and position to save lives.

5 Conclusion

Mathematics does not occur in a vacuum. Mathematics is done by mathematicians, and mathematicians live in human society.

So yes, Hardy was a great mathematician, a great researcher and a great teacher, but he was also a great human being, and today, we all live in his shadow, be it when we take Tripos, or read books about Analysis, or simply live in the international mathematical community. For all that Hardy saw mathematics as profoundly useless, he was still able to use it to change some small part of the world for the better.

Thank you, ladies and gentlemen. Has anyone got any questions?

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